WWW.JBDAI.ORG

ISSN: 2692-7977

JBDAI Vol. 3, No. 1, 2025 DOI: 10.54116/jbdai.v3i1.29

Multinational Investment under Uncertainty

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ABSTRACT

This paper builds a real options model to quantify multinational investment timing decisions under both foreign market demand and exchange rate dynamics, which are largely overlooked in academia yet very common in the real world of international investments. We find that (1) a domestic firm may prefer to undertake foreign direct investment (FDI) under an exchange rate depreciation environment provided high foreign demand and domestically sourced investment costs. (2) Both exchange rate and demand uncertainties could have either positive or negative impacts on international investments, depending on their correlations and the relative dominance between "real option effect" and "revenue effect." A simple simulation exercise confirms model predictions and shows that generally the impact of demand uncertainty should be more prominent than that of exchange rate uncertainty.

JEL: *F21, G31.*

Keywords: *real options model, demand uncertainty, exchange rate uncertainty, FDI.*

1. Introduction

Multinational investments play an important role in economic growth and over the last several decades, the amount of global foreign investment has steadily increased. The decision to invest abroad however, is not an easy one for a multinational enterprise (MNE). According to the international economics and business literature (Conconi et al. 2016), the process of firm internationalization generally evolves from exporting, to building overseas distribution facilities, and eventually foreign production. An important factor that affects the decision to invest abroad is market demand uncertainty. Given the sunk cost of foreign investment, the minimum market demand required by MNEs to invest abroad, is significantly high. Additionally, the foreign investment decision is also influenced by the variation of foreign exchange rate as it affects the home currency denominated profits, both short-run and long-run. Although in prior literature, scholars have investigated the impact of these two factors—market demand uncertainty and exchange rate variations—on multinational investment, they have so far been studied separately. Few scholars have analyzed their joint impact, particularly in a dynamic environment. This paper tries to fill that gap.

The theoretical model we propose applies the canonical real options framework to multinational investment decisions. Real options theory (ROT) has been applied extensively in research in international economics and management studies.[1] ROT posits that optimal capital investment decision under uncertainty is equivalent to solving the optimal exercise timing of the American-type options, given that the output commodities can be traded across complete markets.

Our model is a two-country model, home (or source) and foreign (or host/destination), with one monopolistic firm headquartered in the home country, producing a homogeneous commodity to serve only the foreign market. The firm begins with exporting to the foreign market and eventually needs to make a decision on when to establish affiliates to produce in the foreign country to serve the local demand there. The timing of overseas investment is a function of exchange rate and foreign demand, both of which vary with time in a random fashion. The exchange rate is defined as the home currency value of foreign currency. The operating mode in both countries is such that the firm can freely adjust its production output (zero or full capacity) in response to the uncertain demand and currency evolution. The MNE manager makes decisions on the operating status, as well as the timing of when production is shifted to the offshore destination. In the benchmark case, we assume both countries have unlimited production capacities and then extend it to asymmetric capacity limits. The model delivers following results.

First, the relation between exchange rate movement and foreign investment depends on how the MNEs source the investment, the relative strength of its currency, and its production status before the foreign investment is undertaken. When the investment is sourced locally in the destination country, appreciating home currency will always accelerate the timing of investment outflow. This is intuitive because a higher value of home currency will make the foreign project more attractive and the investment cost cheaper. However, when the investment is sourced from the home country, the result is mixed. On the one hand, when the home currency is relatively weak, depreciating home currency will have a negative impact on foreign direct investment (FDI) because it increases the advantage of production in the home country. On the other hand, when the home currency is relatively strong, depreciating home currency will have a positive impact on FDI because it increases the cash flow value in the foreign country and the cost of foreign investment will be lower.

Second, foreign demand uncertainty and exchange rate uncertainty have either a positive or a negative impact on cross-border investment, depending on the correlation between the two shocks. When the correlation is positive, both uncertainties may either accelerate or delay the investment timing. On the one hand, a positive correlation indicates comovement of both variations. Since in our model higher exchange rate means lower home currency value, the comovement mitigates the variation of foreign revenues and thereby accelerates the investment. We call this the "revenue channel," which has been documented by Goldberg and Kolstad (1995) as well.[2] On the other hand, the existence of demand and exchange rate uncertainties will also give rise to the "real option" value of entering the foreign market, thereby delaying the entry, i.e., the "real option channel." The final outcome depends on the relative dominance of the two forces. However, when the correlation is negative, both uncertainties will deter FDI flow to the foreign country. This is expected since now the variation of foreign revenue will increase and the real option effect is operative. Finally, our model also shows that the relation between the two types of uncertainties and FDI depends on the operating status of the exporter. In particular, when the exporter is at suspension, both types of uncertainties will delay foreign investment, regardless of the correlation level. This is expected because at this stage the firm does not have cash-flow based assets, thus it will be impacted more by the "real options" channel.

In addition to the theoretical model, we also perform a simulation exercise. The simulation is necessary because it can straightforwardly present the possibility of production status and investments. Our simulation results are consistent the model's trigger-based predictions. More importantly, the simulation also illustrates that the impact of demand uncertainty should be more prominent than that of exchange rate uncertainty on the investment probability using our carefully selected parameters. This observation is in line with Choi and Jiang (2009) and Nguyen et al. (2018).

The paper is organized in the following manner. Section 2 presents the literature review. Section 3 lays out some empirical motivation for modelling work. Section 4 elucidates the assumptions and framework of the dynamic model. It also contains numerical solutions. Section 5 performs a simple simulation exercise based on the model-guided solutions. Section 6 concludes the paper.

[1]For a review of ROT applied in investment decision, see Dixit and Pindyck (1994). For the particular application of ROT in international business, see Chi et al. (2019) and Song et al. (2015) for a comprehensive review. Indeed, several recent empirical papers highlight the impact of real options on the multinationality, such as Aabo et al. (2016) and Belderbos et al. (2019).

[2]Note that Goldberg and Kolstad (1995) has different settings from ours. They define the correlation between two levels while we define it between two random shocks. However, despite the different expressions, the relation between covariance and correlation is generally similar.

2. Literature Review

The determinants of FDI decision have been investigated extensively in recent decades. A comprehensive review of the relation between various factors and FDI can be found in Blonigen (2005). Here we only discuss the literature that relates to the impact of exchange rate and demand dynamics on FDI.

First, some prior work shows that higher home currency value (relative to the destination country) will promote FDI outflow. This result was initially documented in Froot and Stein (1991). The reason is simple: higher home currency value will make the MNE wealthier and therefore it will be more likely to acquire foreign assets. Additionally, a depreciating currency will make foreign country's assets cheaper and more likely to be acquired by global firms. However, our results show that this result is mixed and usually it is insignificant due to the interaction between the stochastic environment and the sources of investment costs.

Second, the positive (or nonlinear) impact of currency volatility on FDI has been explored before. For example, Goldberg and Kolstad (1995) show that when the investors are risk-averse and there is negative correlation between exchange rate and demand shocks, the currency volatility could have a positive impact on the share of foreign production. However, when the correlation is positive, the firm will maintain 100% capacity overseas. Darby et al. (1999) theoretically find that exchange rate variation could either accelerate or delay foreign investments depending on the economic parameters and country types. Jeanneret (2016) explores the channel of firm heterogeneity and shows that MNE with high productivity may take the benefit of currency volatility to engage in FDI. Due to this, he finds a U-shaped relation between FDI and exchange rate volatility.

Third, for (country-level) demand uncertainty and foreign investment, Goldberg and Kolstad (1995) show mixed signs of demand uncertainty on FDI (see their equation A.2a) for risk-averse investors. Empirically, Conconi et al. (2016) study the influence of host countries' uncertainty on horizontal FDI decision using detailed Belgium firms' data. They show that the probability that a firm engages in foreign investment increases with its export experience. In more uncertain destinations, firms delay FDI entry, experimenting longer with exports before establishing foreign affiliates.

So far, Goldberg and Kolstad (1995) have similar setting as our analysis with regard to the joint effect of economic uncertainties (e.g., exchange rate and foreign demand). However, we have several critical differences. (1) We focus on modelling the nominal exchange rate instead of the real rate. Although real exchange rate takes into account the purchasing power and will impact the trade balance, nominal rate is more relevant for firm-level and short-term operations. (2) We build a dynamic model and derive its solutions in the context of timing of irreversible investment while they focus on the FDI production share. (3) We focus on the channel of interaction between economic uncertainties and operating flexibility instead of risk-aversion.

Our paper also contributes to the literature on the theory of foreign investment decisions. Rob and Vettas (2003) investigate optimal share of exports and FDI under growing demand and foreign capacity limits. Similar to our analysis, they consider both the investment irreversibility and capacity underutilization. They focus on the interior solution for the coexistence of exporting and foreign investment. Perhaps most distinct from ours is that they assume the demand either increases or stays put while we allow stochastic movement with uncertainties. Aray and Gardeazabal (2010) and Sung and Lapan (2000) investigate the impact of exchange rate on foreign investment under product market competition. Jeanneret (2016) examines the interaction between MNE's productivity heterogeneity and exchange rate uncertainty on FDI. However, unlike our analysis, these papers focus on either exchange rate uncertainty or demand uncertainty, and not both of them together.

3. Empirical Motivation

Before proceeding to the model part, we briefly present some empirical observations in this section. The goal of this exercise does not attempt to demonstrate any "stylized facts" to be proved by the model because there are significant gaps between the real data and model assumptions (listed in Section 4). For example, we only have country-level observations while the model is based on firm-level. The results from aggregating firms (e.g., country-level) could be different from single firms. Moreover, our data could not differentiate between horizontal and vertical investments while the model focuses the former type. Instead, our goal is to demonstrate the counterintuitive and ambiguous impacts of exchange rate movement and its volatility, and demand volatility on bilateral FDI, and therefore elucidate the importance of modelling firm decisions under multifactors. In what follows, we first discuss data background and then directly go to results summary. All empirical details are located in Appendix A.

3.1. Data Background

We sample bilateral FDI between the U.S. and its seven major FDI partners (United Kingdom, Japan, Canada, Germany, Netherlands, Switzerland, and France). These countries are either the recipients of the highest amount of FDI from the U.S. or have been the largest investors into U.S. We limit our analysis to the top seven partners for the years 1982 to 2018. Of the more than 200 countries investing in the U.S., these seven countries consistently remained as the top sources of FDI into the U.S. for the years considered and contributed about 70% of the total investment every year. Similarly, out of the many destination countries for U.S. FDI, these countries feature among the top 10 and receive about 50% of the total U.S. outward investment.

We focus on the FDI data in the manufacturing industries and exclude others such as financial and utility sectors.[3] Bureau of Economic Analysis (BEA) provides the stock of FDI for the year. We convert it into a flow-based measure by taking the difference in the stock of FDI between two years. We normalize the FDI variable by dividing it by gross domestic product (GDP).

3.2. Results Summary

(1) Appreciating home currency seldom promotes FDI outflow, the only two examples are U.S. to Japan and Germany to U.S.[4] Depreciating home currency sometimes increase FDI outflow. This scenario holds for five pairs: U.S. to Switzerland, U.K. to U.S., Japan to U.S., Switzerland to U.S., France to U.S. The rest of seven pairs show that exchange returns have insignificant impact on the FDI flows.

(2) The exchange rate volatility does not have significant impacts on the FDI decision in most cases. When the impact is significant, e.g., U.S. to Japan and U.S. to Switzerland, the signs are opposite.

(3) The foreign demand volatility, estimated indirectly by the GDP growth volatility, has either negative or insignificant impact. The negative impact holds for only three pairs: U.S. to Japan, U.S. to Switzerland, and Germany to U.S.

Although the empirical exercise is limited to certain country pairs, we can sense that the results are contrary to conventional wisdom in most times. Thus, it will be very necessary to develop a microlevel model to understand how these factors impact international investments.

4. Model Set Up

We assume a two-country (e.g., home and foreign), continuous-time and a partial equilibrium economy. A monopolistic firm whose headquarters is located in the home country starts to serve the foreign market through exporting. It possesses an option to establish affiliates abroad, to serve the local market. To simplify the analysis, we make several assumptions:

(1) The foreign investment is of horizontal nature. The firm at the home country makes foreign investment timing decision to maximize its pre-FDI value in terms of home currency.

(2) The home-based MNE only serves foreign market. It serves neither home country nor other foreign countries.

(3) Offshoring production is considered irreversible within the model.

(4) The firm chooses to produce by either exporting or building affiliates abroad but never adopts both strategies at the same time. This assumption enables us to focus on investment timing decision instead of solving for the optimal share of foreign production, which has been addressed in other papers such as Goldberg and Kolstad (1995) and Rob and Vettas (2003).

(5) The sunk cost of foreign investment is sourced from either home country and priced with home currency as in Jeanneret (2016), or foreign country and priced with foreign currency as in Aray and Gardeazabal (2010). In the latter case, the firm valuates the FDI cost in terms of the home currency. We will compare the results from these two scenarios as robustness check. In the following paragraphs, we introduce the model setup.

[3]We do this because our structural model in the later part is built on capacity investment, which is more consistent with manufactory sector. Such treatment has been widely applied in the past literatures. However, our data also shows that FDI of all industries also present similar results.

[4]Note that we define the exchange rate as the number of foreign currencies per USD for all cases. When U.S. is the "home country," the increase of the exchange return indicates dollar becomes stronger. When U.S. is the foreign destination, the increase of return suggests foreign currencies become weaker.

Table 1: Benchmark parameters calibration.

Symbol	Economic Determination	Value
rh	Risk-free rate of home country	0.04
rf	Risk-free rate of foreign country	0.07
γ	Demand elasticity	1
μ	Expected drift of demand	0
σS	Exchange rate volatility	0.10
$\sigma\theta$	Demand volatility	0.10
τ	Ice-berg transportation cost	0.7
I	Suck cost of foreign investment	1
vh	Production cost at home country	0.5
vf	Production cost at foreign country	0.5

The demand function for the foreign market is determined by the following linear equation:

$$P = \theta - \gamma q \tag{1}$$

here P is the local price in term of foreign currency, γ represents the (constant) elasticity of demand, θ is the foreign demand shock following Brownian motion.[5]

$$\frac{d\theta}{\theta} = \mu dt + \sigma_\theta dZ^Q \tag{2}$$

here μ is the expected growth of foreign demand and σ_θ captures the standard deviation.

The exchange rate,[6] defined by *number of home currency per unit of foreign currency*, evolves in a stochastic fashion under the risk-neutral probability measure ($\mathbb{Q}$ measure), for the domestic investor.

$$\frac{dS}{S} = \left(r_h - r_f\right)dt + \sigma_S dW^Q \tag{3}$$

where rh and rf are the domestic and foreign risk-free rate at which money can be borrowed or lent; σ_S is a positive constant representing standard derivation, and $(Wt)t \geq 0$ is a standard Brownian Motion process. Time is continuous and varies over [0, ∞]. Uncertainty is represented by the filtered probability space $\left(\Omega, F, (F_t)_{t\geq 0}, Q\right)$ over which all stochastic processes are defined. However, we essentially assume that the expected rate of return and standard deviation of exchange rate are constants. This assumption deviates from the real world, which might have a time-varying pattern instead of constants. We do so to facilitate the computation of the model. The derivation for Equation (3) is presented in Appendix B. The correlation coefficient between the variations of foreign demand and exchange rate is denoted as ρ, e.g., $dZdW = \rho dt$. As shown in Table 1, the correlation varies in a wide range among the selected countries, which necessitates its role in our model to explain the empirical facts.

Consider the constant marginal cost of production in home country as v_h and the production cost in foreign country is denoted as v_f, therefore the instantaneous revenue for home production to export (exporter "*X*" status) can be written as

$$\pi_X = (\tau SP - v_h)q = (\tau S\theta - \tau S\gamma q - v_h)q \tag{4}$$

here τ captures the revenue loss caused by the ice-berg type transportation cost and q is the production output. We assume there is no capacity limit, thus the optimal production can be derived by $q_X{}^* = \frac{\tau S\theta - v_h}{2\tau S\gamma}$. Since the optimal quantity cannot be negative, we get the profit flow as

$$\pi_X = \begin{cases} \frac{(\tau S_t\theta_t - v_h)^2}{4\tau S_t\gamma} & \theta_t > \frac{v_h}{\tau S_t} \quad 0 \quad \theta_t < \frac{v_h}{\tau S_t} \end{cases} \tag{5}$$

[5]The linear demand function has been used in Sung and Lapan (2000), Rob and Vettas (2003), Aray and Gardeazabal (2010), and Conconi et al. (2016) mostly for its modelling convenience. Another strand of literature that includes Helpman et al. (2004), and Jeanneret (2016) employs CES aggregate price because they focus on the impact of firm heterogeneity of productivities. The former setting is more suitable and tractable for our paper since our model is built on capacity investment.

[6]The exchange rate here is meant to nominal exchange rate because the model is not explicitly feed into price level or inflation discrepancy. As a matter of fact, there isn't clear boundary between nominal and real exchange rate when modeling the international business, and in most cases, when the commodity price is relatively sticky, both of the two rates are closely correlated and not much different in the econometric perspective (Clark et al. 2004, Section III)

It is noteworthy that the operating/suspension boundary is determined by a nonlinear boundary governed by two factors (θ_t, S_t). This indicates that the exporter will produce only when the demand for the product and the exchange rate are high enough $\theta_t S_t > \frac{v_h}{\tau}$. Notice that whenever the exchange rate is relatively small (i.e., the home currency is strong), domestic production will be more likely to suspend due to the high demand bar to restart. This is intuitive because the strong currency will hurt the export sector.

Similarly, profit flow for foreign affiliates (FDI "*F*" status) can be written as

$$\pi_F = S(P - v_f)q = S(\theta - \gamma q - v_f)q \tag{6}$$

and the optimal output quantity is $q_F{}^* = \frac{\theta - v_f}{2\gamma}$ since the optimal quantity cannot be negative, we get profit flow as

$$\pi_F = \begin{cases} \dfrac{S_t(\theta_t - v_f)^2}{4\gamma} & \theta > v_f \quad 0 \quad \theta < v_f \end{cases} \tag{7}$$

The foreign production process is only dependent on market demand because we assume all products will be sold locally although the MNE's consolidated revenue is still impacted by the exchange rate.

Our goal is to solve for the timing of foreign investment. Following standard protocol, we first present the valuation method for foreign affiliates and then we present the valuation for the exporter.

The value of foreign affiliate V_F is governed by the following partial differential equation (PDE) (the derivation is in Appendix B):

$$\frac{1}{2}\sigma_S{}^2 S^2 \frac{\partial^2 V_F}{\partial S^2} + \frac{1}{2}\sigma_\theta{}^2\theta^2 \frac{\partial^2 V_F}{\partial \theta^2} + \rho\sigma_S\sigma_\theta S\theta \frac{\partial^2 V_F}{\partial S \partial \theta} + (r_h - r_f)S\frac{\partial V_F}{\partial S} + \mu\theta\frac{\partial V_F}{\partial \theta} + \pi_F = r_h V_F \tag{8}$$

Although the equation appears complex, it delivers clear economic sense. The first and second item on the left-hand side capture the instantaneous effect of volatilities of exchange rate and foreign demand on foreign assets' value; the third item evaluates the correlation effect between the two dynamic evolutions; the fourth and fifth term capture the instant effect of expected moving rate and the last term is the profit flow. The equation suggests that the entire instantaneous returns should be equal to the risk-free rate at home currency, which is expected as we employ a risk-neutral valuation. In the following, we introduce appropriate boundary conditions to solve the optimal stochastic control problem.

First, when the foreign demand declines to zero the value of the foreign affiliate in terms of home currency should also be zero, no matter what the exchange rate. This is intuitive because profit flow increases linearly with S_t:

$$V_F(\theta \downarrow 0, S) = 0 \tag{9}$$

Second, when the exchange rate is extremely low, or the foreign currency is very weak, the home currency value of the foreign counterpart also approaches zero, for the same reason as the previous one.

$$V_F(\theta, S \downarrow 0) = 0 \tag{10}$$

Third, when demand is very large, the affiliate will not suspend and the value can be conveniently obtained

$$V_F(\theta \uparrow \infty, S) = \Pi_F \tag{11}$$

here Π_F is the present value of all future profits of the affiliate in a risk-neutral measure and can be written as

$$\Pi_F = E^Q \int_t^\infty e^{-r_h(s-t)}\pi_F ds = \frac{S}{4\gamma}\left(\frac{\theta^2}{r_f - 2\mu - \sigma_\theta{}^2 - 2\rho\sigma_S\sigma_\theta} - \frac{2v_f\theta}{r_f - \mu - \rho\sigma_S\sigma_\theta} + \frac{v_f{}^2}{r_f}\right) \tag{12}$$

Finally, we have another boundary at the upper level of exchange rate, $S \uparrow \infty$. However, it is very hard to quantify this condition with any economic intuition. To mitigate this concern, we heuristically assume that the second-order impact is negligible, and that the profit flow is of first order importance to S_t. The condition can then be written as

$$\frac{\partial^2 V_F}{\partial S^2}(S \uparrow \infty) = 0 \tag{13}$$

The detailed implementation is discussed in Appendix C.

Applying dynamic programming, we evaluate the exporter value, V_X, as

$$V_X(\theta, R) = \{\pi_X(\theta_t, S_t)\Delta t + E[V_X(\theta, S)\Delta t]; V_F(\theta, S) - I\} \tag{14}$$

This equation indicates that the exporter value will be equal to the larger of (1) its own continuation value in the next instant and (2) its net present value upon foreign investment. Note that here we assume that the foreign investment expenditure I is sourced from home country. In the model extension section, we provide an example where the expenditure is sourced locally. In that case, the last item in the parentheses will be $V_F(\theta, S) - SI$. Following standard protocol, we can rewrite it as following PDEs

$$\frac{1}{2}{\sigma_S}^2 S^2 \frac{\partial^2 V_X}{\partial S^2} + \frac{1}{2}{\sigma_\theta}^2 \theta^2 \frac{\partial^2 V_X}{\partial \theta^2} + \rho\sigma_S\sigma_\theta S\theta \frac{\partial^2 V_X}{\partial S \partial \theta} + \left(r_h - r_f\right) S \frac{\partial V_X}{\partial S} + \mu\theta \frac{\partial V_X}{\partial \theta} + \pi_X = r_h V_X \tag{15}$$

To solve the equation, we need a series of boundary conditions:

First, similar to the valuation of foreign affiliates, when foreign demand shock is extremely low, the firm value approaches zero, irrespective of the exchange rate.

$$V_X(\theta \downarrow 0, S) = 0 \tag{16}$$

Second, when the exchange rate is extremely low, the firm will have no incentive to serve the foreign market because the demand bar for production will amount to infinity. This is intuitive since a strong home currency leads to weaker exports.

$$V_X(\theta, S \downarrow 0) = 0 \tag{17}$$

Finally, it is noted that the boundaries at upper levels of θ and S do not have clear economic intuition to characterize. We simply assume the third-order impact is negligible since the profit function is up to the magnitude of second-order. The detailed implementation of the conditions in the finite-difference scheme is discussed in Appendix B.

$$\frac{\partial^3 V_X}{\partial S^3}(S \uparrow \infty) \to 0 \tag{18}$$

$$\frac{\partial^3 V_X}{\partial \theta^3}(\theta \uparrow \infty) \to 0 \tag{19}$$

In the next part, we discuss the solution to the differential equation system.

4.1. Numerical Solution

We calibrate the parameters input to a mix of the ones being used extensively in past literature and from empirical data. For some of structural parameters, where the values are either new to the model or missing in literature, we simply take the best estimates.

4.1.1. Economic parameters

The risk-free rate of return of home country r_h: Jeanneret (2016) calibrated a dynamic model of sovereign debt and obtained a risk-free rate for emerging countries as 4.46%, in line with the average 10-year U.S. treasury rate; the 10-year yield of German government bond is 3.52%. Moreover, Jeanneret (2016) adopts a rate of 3.5% in a multinational investment model in a similar real options framework. Given the discrepancy in the model settings we adopt an average of r_h = 4% without loss of generality. The value loss of iceberg type exporting cost is set to τ = 0.7, consistent with Jeanneret (2016) and Fillat and Garetto (2015). We do not have an accurate measure of this cost and most literature treat it arbitrarily since it does not alter the main implication only the magnitude.

For the standard deviation of demand shock embedded in the Brownian motion σ_θ, Fillat and Garetto (2015) set the value as small as 0.022 to match the U.S. aggregate consumption uncertainty. However, since the direct measure of the standard deviation of real GDP is quite small to generate firm dynamics, Garetto et al. (2018) employ a new method. They calibrate a variety of countries and show that the standard deviation of real GDP ranges from 0.116 (U.S.) to 0.144 (Ireland). To accommodate those differences, we take a moderate level of 0.1. The standard deviation of foreign exchange rate, σ_S is set to 0.1, which is set close to the country average in Jeanneret (2016, online Appendix). In his sample, most developed countries are in the range of 4–7%. We take a relatively high exchange rate volatility to highlight the qualitative property. A lower value will not alter the results.

For the expected growth of demand shock μ: note that this value is constrained by a caveat of the dynamic model under risk-neutral measure. This is because the risk-adjusted discount rate should be positive or it leads to asset bubbles. A simple rationale can be found in Equation (12). To ensure the denominator is positive, we have to require $2r_h > r_f > 2\mu + {\sigma_\theta}^2 + 2\rho\sigma_S\sigma_\theta$, which means that the expected consumption growth should not be too large. For instance, given the value of other parameters, μ has to be less than 0.015. To facilitate computation, we set μ = 0.

However, it is still a reasonable level within [0, 1.5%] as it falls in this range in most dynamic corporate finance literature.

The sunk cost and the production cost of international investment are structural parameters, which lack empirical support. We simply set $I = 1$. Theoretically, it should not impact our results because it is a normalized value. The production cost at home and foreign countries are set to 0.5, i.e., $v_h = v_f = 0.5$. The value of production cost varies greatly in prior literature but such variation will mostly not impact our main results. Table 1 summarizes all parameters the model uses.

4.2. Results Discussion

In this section, we discuss solutions to our benchmark model. We will focus on the impact of exchange rate movement, exchange rate volatilities and foreign demand volatilities on foreign investment decision. We begin with the general shape of the investment threshold decision as a function of both demand and exchange rate movement. In Figure 1, the solid line represents the investment boundary, which can be characterized as a function $\theta(S)$, i.e., demand level varying with the exchange rate. It captures the lowest demand bar (as a function of exchange rate) across which the exporter shift production to foreign affiliates. Given the firm has not yet undertaken FDI, the dotted line represents the demand threshold, which varies with the exchange rate. The level of exchange rate separates the two operating stages, e.g., suspension and production. The figure clearly shows that exporters will conduct overseas investment if the foreign market demand is sufficiently large, irrespective of the exchange rate. This is consistent with our intuition.

For a more detailed understanding of the nonlinearity of the investment boundary, we study four different types of firms, labeled R1 ~ R4. Firms in region R1 and R2, where the demand level is below ~1.3, the exporter will never engage in foreign investment unless the foreign demand increases. It will only change between the suspension and production statuses as the exchange rate varies. In this case, the exchange rate will have no impact on the international investments because demand is too weak. It can also be observed that a weaker home currency will have a positive impact on the exporter so that the production status will be expected, while strong home currency will more likely leave the firm at idling status.

Notice that the foreign revenue will be converted to home currency while sunk cost is directly priced with home currency as we assumed. It means that the foreign revenue will fluctuate with exchange rate while sunk cost will

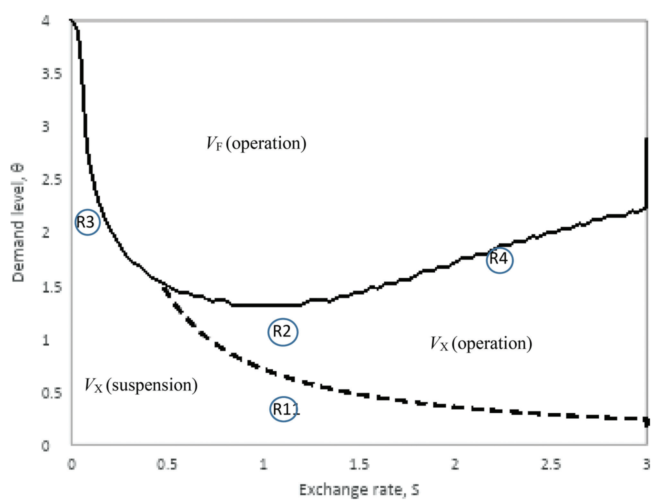


Figure 1: FDI threshold with demand and exchange rate variation. The solid line represents trigger of international investment as a function of (S_t, θ_t). The dotted line represents suspension/operation switching trigger for exporter-type firm, note that the switching threshold is also a function of both demand and exchange (S_t, θ_t). The suspension area lies to the left of switching trigger while operation area lies in the right side. The area above investment threshold belongs to foreign production. The parameters are: correlation between volatilities of demand and exchange ρ = 0.4, volatilities of demand and exchange rate are, respectively, $\sigma_\theta = 0.1$ and $\sigma_s = 0.1$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency.

not. In this case, if home currency is very expensive, then it will hurt the NPV of foreign investment. NPV = $SV_F - I - V_X$, where S is the home currency value of foreign currency and I is the investment cost directly measured in home currency. For example, for the firm type R3, the propensity to undertake FDI will increase as the home currency becomes weaker. This result might seem to be in conflict with conventional wisdom since weaker currency seems to promote exporting rather than FDI. This happens because firms like R3 have no cash flow-based assets, only real options to either go abroad or produce at home. As the home currency depreciates, the value of "investment option" to engage in the foreign investment dominates the "restart option" to export at full capacity. However, when the home currency is too weak, the firm is better off producing and exporting, since the benefits from FDI diminishes, making the investment curve convex.

Similarly, when the home currency is relatively weak and the demand is mildly high, the firm will be an exporter with production status, which is the firm R4. In this case, the firm is more likely to engage in international investment as the home currency become stronger (imagine that S_t moves left), consistent with conventional wisdom.

To better visualize the exporter value as a function of market demand θ and exchange rate S, we plot a three-dimensional figure as well as a contour map in Figure 2. It can be clearly seen that as both S and θ become smaller the firm value approaches zero. This is not surprising since the home currency is very strong and foreign demand is very low. Moving far from (0,0) away to the northeast direction, i.e., as demand becomes very strong and exchange rate very weak, the firm will immediately start foreign investment since it has been deep-in-the-money. It explains

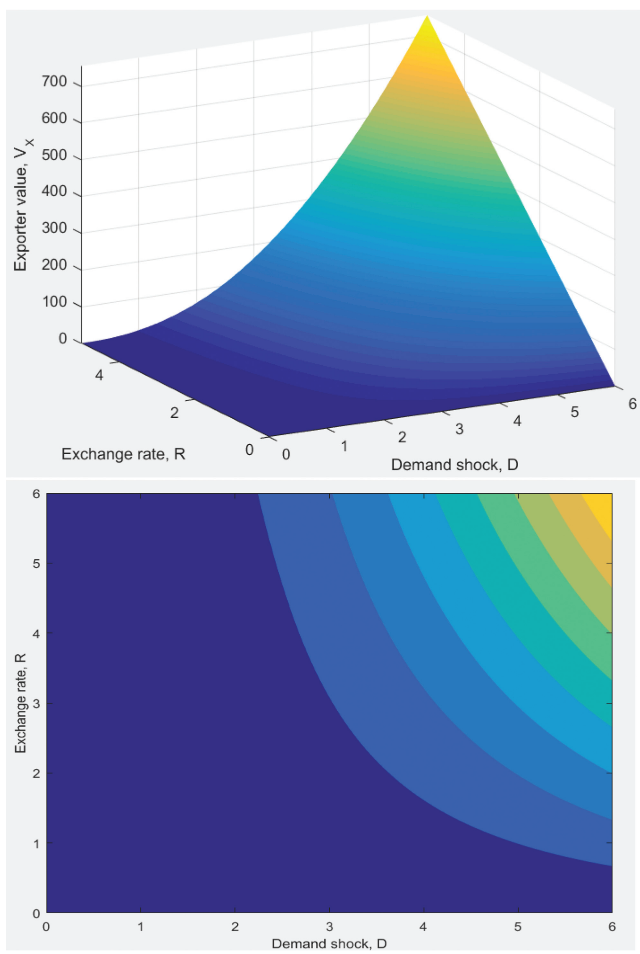


Figure 2: Firm value for exporter. The left panel plots market value of exporter status as a function of (S_t, θ_t) and the right panel plots from a two-dimensional contour view. The parameters are: correlation between volatilities of demand and exchange rate $\rho = 0.4$, volatilities of demand and exchange rate are, respectively, $\sigma_\theta = 0.1$ and $\sigma_s = 0.1$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency.

why a positive correlation can make demand or exchange rate uncertainties benefit investment decisions. We will leave the discussion of correlation to the next part.

In this section, we discuss the impact of demand and exchange rate volatilities (σ_θ and σ_S) and their correlations (ρ) on the trigger values. Figure 3 graphs the impact of demand volatilities on the investment trigger. When the correlation is positive, it can be observed that higher demand volatility may accelerate investment timing for a certain range of exchange S (for example, when S is relatively large). Notice that a larger V_F area represents a higher possibility to hit the foreign investment decision.

The result may look counterintuitive since it is conventional wisdom that higher demand volatility should deter investment timing. The intuition behind this is that positive correlation will make the post-investment foreign assets more valuable, as in Equation (12). This effect will be more prominent given the positive standard deviations of either demand or exchange rate. We call this the "revenue channel." Additionally, higher demand uncertainty will also deter investment due to the "real option" effect. When the revenue effect dominants the real option effect, the MNE will accelerate the investment. When the correlation is negative, demand uncertainty always delays entry, since the foreign asset value will be discounted more heavily. However, it needs to be noted that the investment trigger is defined by two-dimensions. Therefore, a simulation exercise (Section 5) would help to visualize the impact.

More interestingly, when the exporter is at suspension (exchange rate $S_t \sim <1$), higher demand uncertainty ($\sigma_\theta = 0.1$) will delay foreign investment, irrespective of the sign of correlation. This is expected because at this stage the firm does not have any revenues but has an investment option (to enter foreign market) or a restart option (to restore production for export). In this case, the real option effect will play a more important role.

Figure 4 graphs the impact of exchange rate uncertainty on FDI decision. In general, we can observe that the results are similar to that of demand uncertainty. When the correlation is positive, the impact of exchange rate uncertainty is ambiguous, when the correlation is negative, higher volatility always delays foreign investments. The reasoning is similar to that of demand uncertainty so we will not repeat here. It will be valuable to compare our results with Goldberg and Kolstad (1995). In their paper, under positive correlation, the firm always allocates 100% capacity overseas since that lowers the profits variance. Although we apply the same logic, their model cannot predict the investment decision. Under negative correlation, they (proposition 3, page 864) show that exchange rate volatility will have a positive impact on the share of foreign production, however, their model cannot draw any conclusion on the absolute FDI level. In this sense, we are looking for different aspects of FDI and are not in conflict with their results.

In the next part, we introduce two important extensions that are relevant to the real economy. We repeat similar analysis as before.

4.3. Extension 1: Alternative Sunk Cost Source

In this section, we present solutions to the model with alternative sunk cost of foreign investment, that is, the investment cost is composed of materials and technologies bought locally. Consequently, the decision of firm owners can be rewritten in a dynamic programming fashion

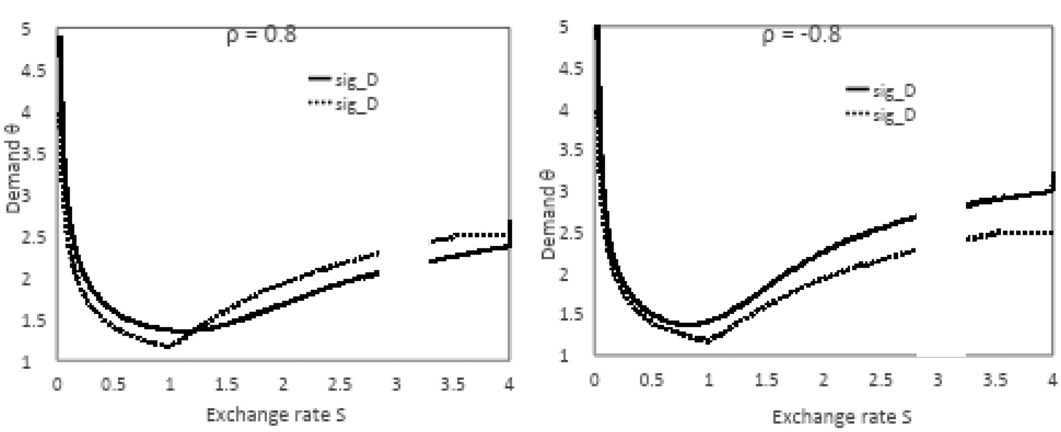


Figure 3: The impact of demand uncertainty on FDI. The figure depicts the impact of demand uncertainty on FDI for low demand volatility (dotted line, $\sigma_\theta = 0$) and high demand volatility (solid line, $\sigma_\theta = 0.1$). The exchange rate volatility is set as $\sigma_s = 0.2$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency. The demand elasticity $\gamma = 1$.

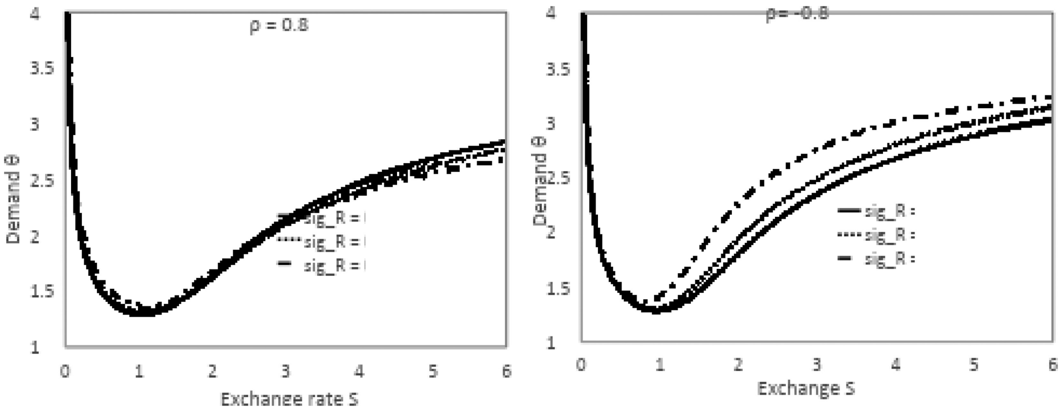


Figure 4: The impact of currency volatility on FDI. The figure depicts the impact of currency volatilities on FDI for positive correlation (left) and negative correlation (right). The solid lines correspond to the case of low currency volatility while the dotted line represents high volatility. The parameters are: volatilities of demand is $\sigma_\theta = 0.1$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency.

$$V_X(\theta, R) = \{\pi_X(\theta_t, S_t)\Delta t + E[V_X(\theta, S)\Delta t]; V_F(\theta, S) - SI\} \tag{20}$$

It is similar to Equation (14) and the only difference lies in the last item *SI*.

Figure 5 demonstrates the investment threshold as the function of demand, depending on the exchange rate levels. In contrast to the benchmark case, the foreign investment threshold presents monotonic relation with exchange rate. Foreign investment is postponed as exchange rate becomes weaker. At the suspension region, the demand trigger for foreign investment is independent of exchange rate. This is intuitive since now the sunk cost is priced in local currency and needs to be converted to home currency. In empirics, this figure suggests that appreciating home currency always has a positive impact on investment in the foreign countries and there will not be any nonlinearity.

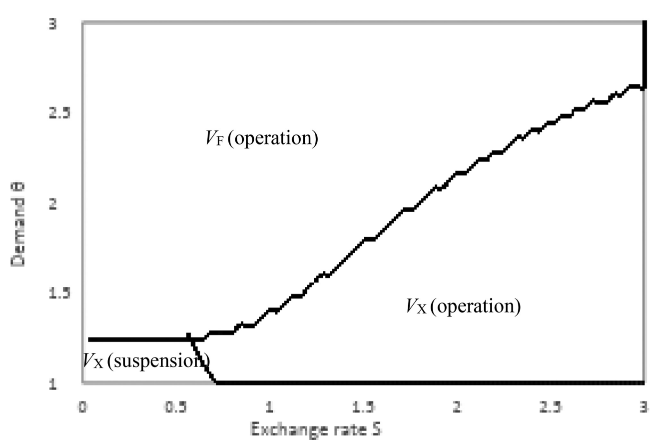


Figure 5: FDI threshold with demand and exchange rate variation. The solid line represents trigger of international investment as a function of (S_t, θ_t). The dotted line represents suspension/operation switching trigger for an exporter-type firm. The suspension area lies to the left of switching trigger while operation area lies in the right side. The area above investment threshold belongs to foreign production. The parameters are: correlation between volatilities of demand and exchange $\rho = 0.4$, volatilities of demand and exchange rate are, respectively, $\sigma_\theta = 0.1$ and $\sigma_s = 0.1$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency.

Figures 6 and 7 plot the impact of demand volatility and exchange rate volatility, respectively. It can be seen that the results pattern hold for an alternative sunk cost. Since the mechanisms are the same as the benchmark case, we will not repeat it here.

4.4. Extension 2: Asymmetric Capacity Limit

In this section, we present a solution to the case with asymmetric capacity at home and foreign. It is probably the most common scenario around the world. Without loss of generality, we assume the domestic production limit is $\underline{Q}_X$ and foreign is $\underline{Q}_F$ and $\underline{Q}_F > \underline{Q}_X$. This extension will be more appropriate for market seekers such as FDI outflow from other countries to U.S. We can rewrite the profit function for the exporter as follows.

$$\pi_X = \begin{cases} \left(\tau S\theta - \tau\gamma S\underline{Q}_X - v_h\right)\underline{Q}_X & \theta > 2\gamma\underline{Q}_X + \dfrac{v_h}{\tau S} \\ \dfrac{(\tau S\theta - v_h)^2}{4\tau S\gamma} & \dfrac{v_h}{\tau S} < \theta < 2\gamma\underline{Q}_X + \dfrac{v_h}{\tau S} \\ 0 & \dfrac{v_h}{\tau S} > \theta > 0 \end{cases} \tag{21}$$

And similarly for post-FDI profit

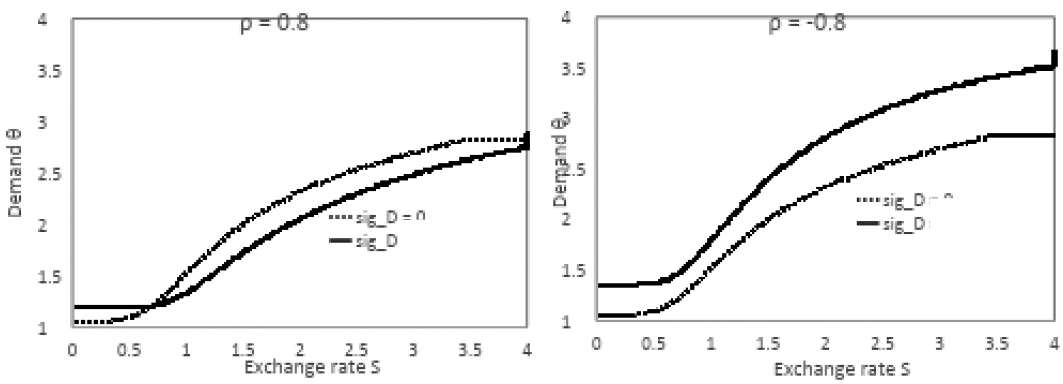


Figure 6: The impact of demand uncertainty on FDI. The figure depicts the impact of demand uncertainty on FDI for low demand volatility (dotted line, $\sigma_\theta = 0$) and high demand volatility (solid line, $\sigma_\theta = 0.1$). The exchange rate volatility is set as $\sigma_s = 0.2$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency. The demand elasticity $\gamma = 1$. In particular, the FDI expenditure will be sourced locally.

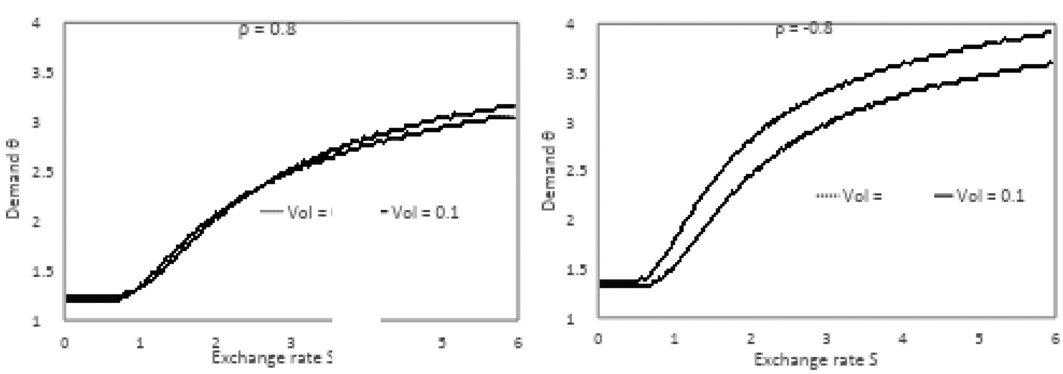


Figure 7: The impact of currency volatility on FDI. The figure depicts the joint impact of correlation and currency volatilities on FDI for positive correlation (left) and negative correlation (right). The solid lines correspond to the case of low currency volatility while the dotted line represents high volatility. The parameters are: volatilities of demand is $\sigma_\theta = 0.1$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency.

$$\pi_F = \begin{cases} S\left(\theta - \gamma \underline{Q}_F - v_f\right)\underline{Q}_F & \theta > 2\gamma \underline{Q}_F + v_f \\ \frac{S\left(\theta - v_f\right)^2}{4\gamma} & v_f < \theta < 2\gamma \underline{Q}_F + v_f \\ 0 & v_f > \theta > 0 \end{cases} \tag{22}$$

We define the capacity limit trigger at home country as $\underline{\theta}_h = 2\gamma \underline{Q}_X + \frac{v_h}{\tau S}$ and for the foreign country as $\underline{\theta}_f = 2\gamma \underline{Q}_F + v_f$. It can be observed that the order of the two depends on the exchange rate dynamics S_t as well as the relative magnitude of the capacity limit, and variable costs. The foreign capacity limit, however, is independent of both exchange rate and exporter cost.

Figure 8 presents the investment decisions as a function of demand and exchange rate. It can be observed that the general shape is similar to that of Figure 1, except for an additional exporter status, i.e., underutilized. There are two interesting observations here: (1) the chances of being in the underutilized status is seemingly higher than the other two; (2) the exporter will always begin to conduct foreign investment when the foreign capacity is at underutilized status. This is intuitive because investment in full foreign capacity may require higher demand threshold and appropriate exchange rate, causing longer waiting time. Meanwhile, investment at underutilized capacity will maximize firm value *ex ante*.

Figure 9 graphs a 2-D and 3-D view of exporter value. The general shape is similar to Figure 2. Larger foreign demand and exchange rate (weaker home currency) will make the home-based firm more valuable. However, it is noteworthy that the highest value occurs not necessarily at the highest demand level (D ∼ = 5).

Figure 10 and Figure 11 plot the impact of demand volatility and exchange rate volatility on the FDI decision, respectively. It can be seen that the results still hold for alternative capacity settings.

5. Simulation Exercise

So far, our model has delivered numerical results for foreign investment decision under both foreign demand and exchange rate uncertainties. However, the model only presents time-invariant (stationary) decisions while the real firms operate in a dynamic world. Second, the timing decisions are hard to interpret with the empirical data, especially given our two-dimensional background. We need to convert the trigger decision to an intensive margin. In the next paragraphs, we first attempt to address some parameter generation process then we proceed to quantitative analysis.

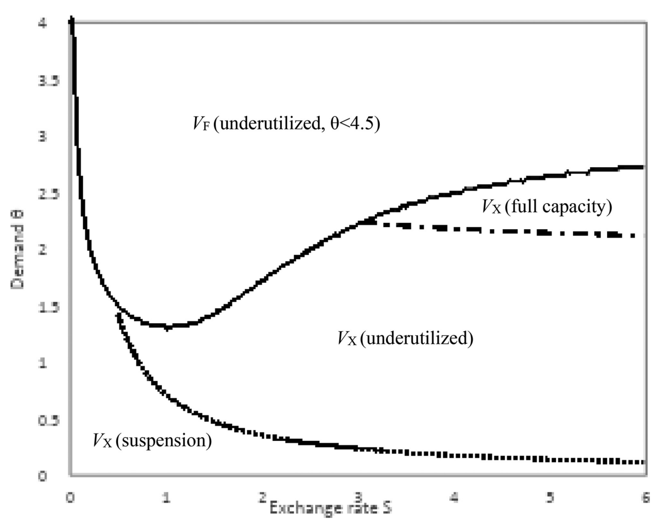


Figure 8: FDI threshold for limited capacity. The solid line represents trigger of international investment as a function of (S_t, θ_t). The dotted line represents suspension/underutilization switching trigger for exporter-type firm, the dot dashed line represents boundary between underutilization and full capacity production. The area above investment threshold belongs to foreign production below full capacity. The parameters are: correlation between volatilities of demand and exchange $\rho = 0.4$, volatilities of demand and exchange rate are, respectively, $\sigma_\theta = 0.1$ and $\sigma_s = 0.1$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency. The capacity limit of home country is $Q_h = 1$ while that for foreign country is $Q_f = 2$.

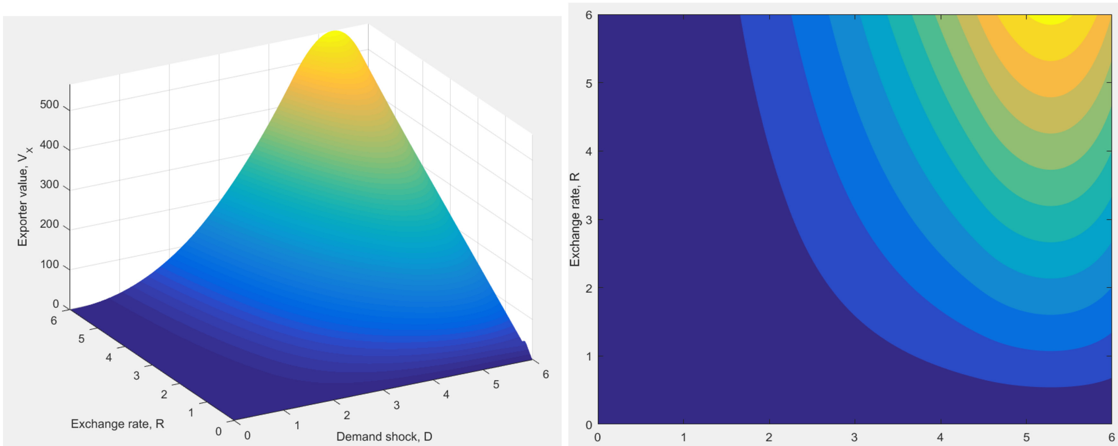


Figure 9: Exporter value. The left panel plots market value of exporter status as a function of (S_t, θ_t) and the right panel plots from a 2-D contour view. The parameters are: correlation between volatilities of demand and exchange $\rho = 0.4$, volatilities of demand and exchange rate are, respectively, $\sigma_\theta = 0.1$ and $\sigma_s = 0.1$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency. The capacity limit of home country is $Q_h = 1$ while that for foreign country is $Q_f = 2$.

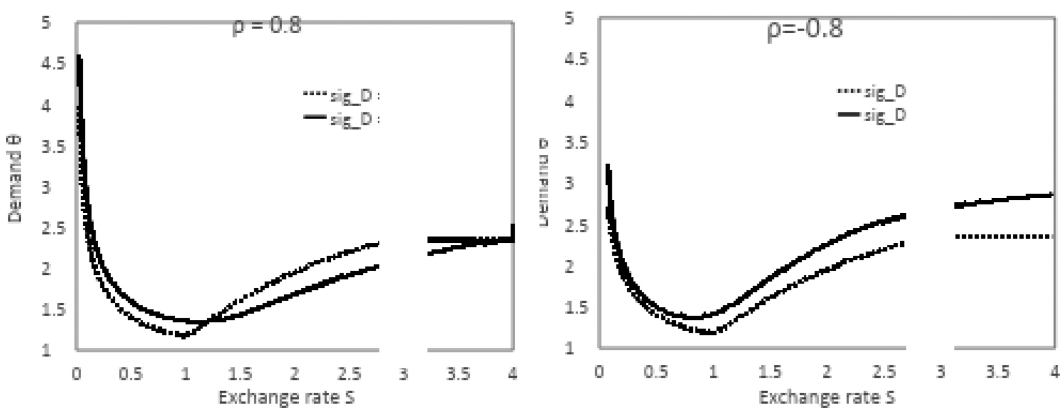


Figure 10: The impact of demand volatility on FDI.

5.1. Price Dynamics and Parameters Setting

The simulated economy is also composed of one home country and one foreign country. Consistent with the model, we simplify the economy so that there is only one firm in the home country and it will either export or make direct investment in the foreign market. In this regard, our simulation will neglect firm heterogeneity (e.g., firm level parameters). The economy is simulated in a quarterly frequency $\Delta t = 1/4$ over 50 years.

We first express the dynamics of foreign demand and exchange rate in an explicit form, i.e., obtain the solution to stochastic differential equations (SDEs) of Equation (2) and Equation (3), given that there is correlation between the two processes. For instance, in the special case of zero correlation between the demand and exchange rate shock, $\rho = 0$. We can obtain the exact solution to the equation since they are log-normal distribution, in particular, the price flow can be discretized as follows:

$$\theta_t = \theta_0 \left[exp\left(\mu - \frac{1}{2}{\sigma_\theta}^2 \right) t + \sigma_\theta \sqrt{t} \sum_{j=0}^{t} Z_j \right]$$

and the exchange rate can be discretized as follows:

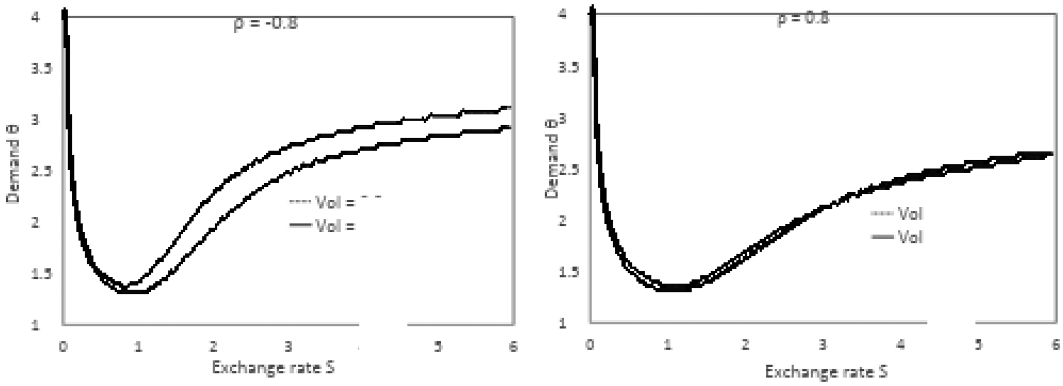


Figure 11: The impact of currency volatility on FDI. The figure depicts the joint impact of correlation and currency volatilities on FDI for positive correlation (left) and negative correlation (right). The solid lines correspond to the case of low currency volatility while the dotted line represents high volatility. The parameters are: correlation between volatilities of demand and exchange $\rho = 0.4$, volatilities of demand and exchange rate are, respectively, $\sigma_\theta = 0.1$ and $\sigma_s = 0.1$, the expected growth rate of demand $\mu = 0$. The discount rate at home country and foreign country are $r_h = 0.04$ and $r_f = 0.07$, respectively. The FDI cost is $I = 1$ in terms of home currency. The capacity limit of home country is $Q_h = 1$ while that for foreign country is $Q_f = 2$.

$$S_t = S_0\left[exp\left(r_h - r_f - \frac{1}{2}\sigma_\theta{}^2\right)t + \sigma_S\sqrt{t}\sum_{j=0}^{t} W_j\right]$$

However, whenever $\rho \neq 0$, there is no closed form solution because the two Weiner processes share a covariance matrix, $Corr(dZ, dW) = \rho$ therefore we need to generate

$$d\Omega = \begin{pmatrix} dZ \\ dW \end{pmatrix} \sim N(0, \Lambda)$$

and the covariance matrix is

$$\Lambda = \begin{pmatrix} \Delta t & \rho\Delta t \\ \rho\Delta t & \Delta t \end{pmatrix}$$

Since there are no closed form solutions for the discretization, we opt for a numerical solution to the SDEs, in particular, under Ito-Tylor expansion and keeping only order-1 items we obtain *Euler* scheme

$$\theta_t = \theta_{t-1} + \mu\theta_{t-1}\Delta t + \sigma_\theta\Delta\tilde{Z}$$
$$S_t = S_{t-1} + \mu S_{t-1}\Delta t + \sigma_S\Delta\tilde{W}$$

The new unknown Weiner process $\Delta\tilde{Z}$ and $\Delta\tilde{W}$ have no analytic expression but they have to obey the above covariance matrix. In order to do this, we calculate L knowing that $\Lambda = LL^T$ and simulate $D\psi \sim N(0, I^2)$ to obtain $d\Omega = Ld\Psi$. We use the quadratic resampling method to generate $D\psi$, which will affect $d\Omega$ by the way we construct. Let us define $E\Psi$ and $\Lambda\Psi$ as the theoretical mean and covariance matrix of $D\psi$, respectively, namely,

$$E\Psi = (0\ \ 0\ \ ) \text{ and } \Lambda\Psi = (1\ \ 0\ \ 0\ \ 1\ \ )$$

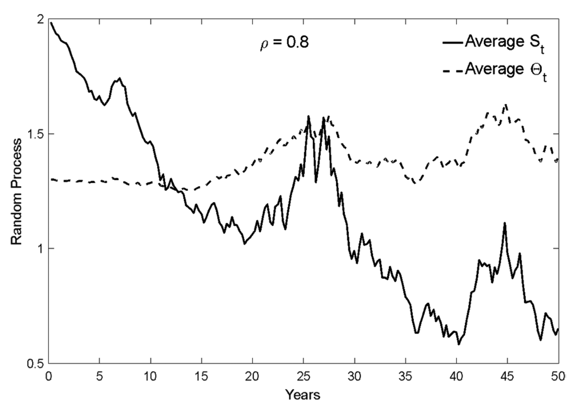
In our model, the investment timing decision is captured by two underlying variables, that is, *trigger* = *trigger* (S, θ). We wish to transit it to investment intensity (e.g., probability measure) in a simulated panel. In particular, at $t = 0$ we assume the MNE is born in the home country. The combination (θ_0, S_0) is too low to allow this firm to engage in overseas investment. However, it will keep the firm as an exporter at production status. As the two variables evolve with time, the firm may serve the foreign market through either exporting or foreign direct investment, or suspension as an exporter. For example, according to the solution in Figure 1, the contour map shows that there are three regions neighboring the threshold line. The firm status controlled by (θ_t, S_t) in the simulated economy will be mapped to corresponding regions contained in the theoretical solution. Whenever the FDI event is triggered, the home-based MNE will engage in the investment in the foreign market and meanwhile, the existing MNE is "retired" and replaced by a newborn MNE in the home country. The new MNE is endowed with the same operating parameters.

There is another caveat in simulating the two-dimensional stochastic control problem. The PDE model only delivers a limited range of solutions with pre-determined stepwise in the finite-difference scheme for $\theta(S)$ thus its calculation is time-consuming. In this case, for any simulated exchange rate S_t outside the model range we interpolate these new points to fit the nonlinear curve. This method not only allows solutions for any possible simulated value of exchange rate S_t but also makes the solutions more continuous and thus smooths out the curve.

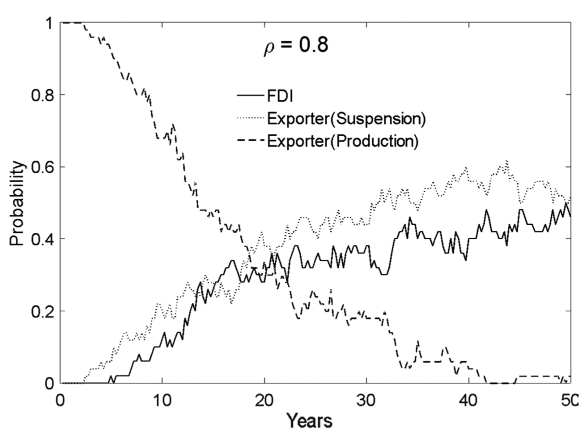
To conduct quantitative analysis of simulated foreign investments, we focus on simple graphic presentations instead of regression analysis. This is because regression analysis demands matching moments between simulated and empirical data. In our case, since we focus on a simple two-country economy, producing data moments in time series is more important than in the cross-section. Unfortunately, due to the curse of model dimensions, we have to limit the model capacity up to stationary (time-invariant) solutions and we exclude time dependent returns and volatilities. The lack of time-series accuracy lowers the validity of regression results. Second, in a random simulation, a relatively large batch of samples is required to generate a robust result. However, this protocol may even weaken the impact of economic uncertainties especially when we average the sample to get expected investment intensity, losing our main purpose. As a consequence, our simulation exercise is not intent to perform empirical specification, instead it tries to validate intuitions reflected from previous bi-dimensional figures. In the following, we introduce a series of simulated results.

5.2. Exhibition 1: The Impact of Exchange Rate (Returns) on International Investment Decisions

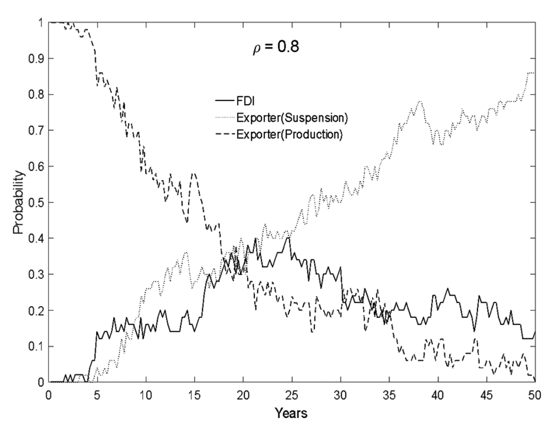
In Figure 12(a), we present the simulated diffusive process of exchange rate S_t (solid line) and foreign demand θ_t (dashed line). They are obtained by averaging over 50 simulated samples following previous construction protocol.



(a)



(b)



(c)

Figure 12: (a) A sample path of (S_t, θ_t) with the correlation of Winer process $\rho = 0.8$. The parameters are selected as $N = 100$, $\theta_0 = 1.3$, $S_0 = 2$, $\rho = 0.8$, $\sigma_S = 0.2$, $\sigma_\theta = 0.1$. The value of each randomness is calculated by averaging the 100 paths. (b) Depicts the production status for a MNE whose foreign investment cost is sourced locally. (c) Depicts the production status for a MNE whose foreign investment cost is sourced from home country.

We choose small samples to average to avoid eliminating volatilities as mentioned before. The starting point is set at $\theta_0 = 1.3$ and $S_0 = 2$ (R4 firm type in the Figure 1), so the exporter is at production status and has not engaged in the foreign investment. The correlation of shocks is $\rho = 0.8$. It can be observed that the volatilities of the two process comove together, consistent with the positive correlation. The exchange rate decreases amid ups and downs due to the negative drift (4% − 7% = −3%). The demand process does not deviate much since the drift is set to zero.

Figure 12(b) and 12(c) present an MNE's production status as the exchange rate and foreign demand evolve with time. The difference between them lies in the assumption of cost structure of foreign investment expenditure. Panel (b) is for the case that the investment expenditure is expensed at the local currency. The probability associated with each production/investment status is computed by counting the frequency of each happenings scaled by total simulated paths. Recall that we assume the MNE is initially an exporter with active production. Consistent with the intuition from Figure 5, the chances of both engaging in FDI and suspending current home production increase as the home currency appreciates (as shown in Figure 12(a)). Panel (c) is for the case that the expenditure is sourced from home country, i.e., our main model. Recall in the Figure 1, the numerical solution shows that if the firm is initially at an active exporter status (say R4 type), then appreciating home currency will initially increase and then decrease the possibility of FDI while the status of suspending production become more likely. The simulation result in Panel (c) confirms the general pattern.

5.3. Exhibition 2: The Impact of Demand Volatility on International Investment

Our next focus is the impact of foreign demand uncertainty. Since as we have seen previously this result is robust for all types of scenarios, we just select the case in the Extension 1 as illustration. Our numerical solution illustrates that positive correlation between the demand and currency dynamics may produce positive impact of the demand uncertainty on foreign investment, the result is confirmed in Figure 13(left panel). Higher demand uncertainty causes accelerated FDI entry compared to the case with lower demand uncertainty, but this trend is reversed given sufficiently long period of time. This happens because of the relative dominance of "real option effect" and "revenue effect," as discussed before. Again, when the correlation is negative, high uncertainty always has negative impact on foreign investments because only "real option effect" remains.

5.4. Exhibition 3: The Impact of Exchange Rate Volatility on International Investment

Figure 14 plots the FDI decision under the impact of exchange rate variation. Similar to demand uncertainty we find that positive correlation leads to some positive or insignificant impact of high exchange rate uncertainty (left panel) while negative correlation usually entails negative impact of high currency volatility (right panel), particularly if we neglect the first 20 years. However, notice that the results are not so pronounced compared to the demand factor. This result is interestingly consistent with other empirical findings where the exchange rate volatility generally has insignificant impact on the FDI decision (see Choi and Jiang (2009) and Nguyen et al. (2018)). Such a coincidence can be attribute to the dominant impact of foreign demand relative to the that of exchange rate in our model framework.

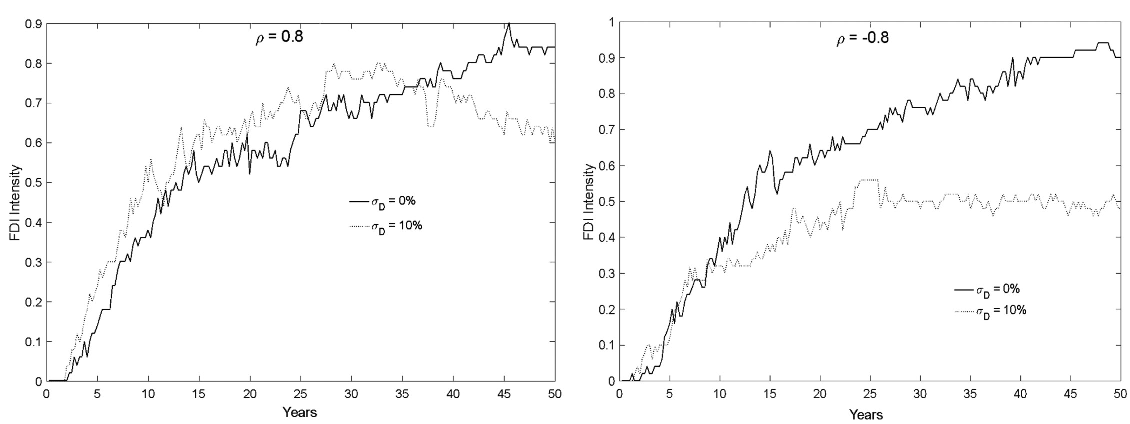


Figure 13: Depicts the impact of demand uncertainty on the FDI intensity for *positive correlation* (left) and *negative correlation* (right). The parameters are selected as $N = 100$, $\theta_0 = 1.5$, $S_0 = 3$, $\sigma = 0.2$. The solid line depicts the case of low demand volatility (0%) and the dotted line is for high demand volatility (10%).

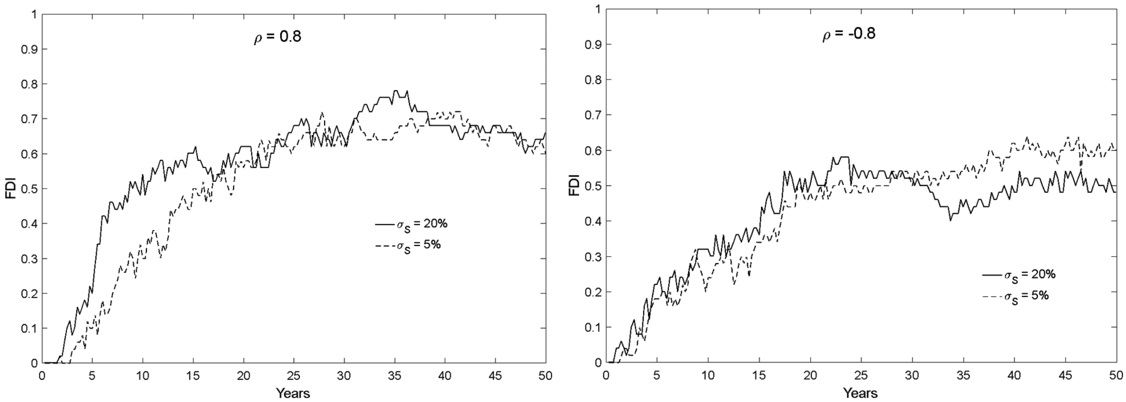


Figure 14: Depicts the impact of exchange rate uncertainty on the FDI intensity for positive correlation (left) and negative correlation (right). The parameters are selected as $N = 100$, $\theta_0 = 1.5$, $S_0 = 3$, $\sigma_D = 0.1$. The solid line depicts the case of high exchange rate volatility (20%) and the dotted line is for low exchange rate volatility (5%).

Additionally, Goldberg and Kolstad (1995) theoretically show that the positive correlation should increase the overseas production share. In our simulation exercise, we observe that the role of correlation has to be interpreted with the help of other factors such as demand or exchange rate volatilities. For instance, in the Figure 13, the positive correlation has nearly zero impact for the case of low demand volatility whereas it elevates FDI in the case of high demand volatility. In Figure 14, the positive correlation seemingly improves FDI for both high and low exchange rate volatilities. Therefore, the effect of correlation requires large resampling batches from simulations to construct statistical inference. However, this is beyond the scope of this paper, given our bidimensional setting.

6. Conclusion

In this paper we build a dynamic continuous-time model to characterize the (horizontal) foreign investment decision in the presence of both stochastic exchange rate and stochastic foreign demand. We show that exchange rate depreciation could have either positive, negative, or insignificant impact on FDI, depending on the source of the irreversible cost. If the investment expenditures are sourced in the destination country, then the appreciation of home currency always has positive impact on FDI; if the investment expenditures are sourced from home country, then there is a nonlinear relation between exchange rate return and foreign investment. Given in the real world, both cases exist, the aggregate results could vary.

We also show that both exchange rate and foreign demand volatility also could have mixed impact on FDI, there are two forces behind this phenomenon. First, the volatility itself can introduce real option effect that postpones foreign investments; Second, the revenue effect that makes the foreign affiliates more valuable, accelerates foreign investments. In empirical sense, the exact direction of uncertainty influence depends on the relative dominance between the two forces.

Our model contributes to the literature and encourages more insightful research in the future. In empirics, a larger dataset of firm-level foreign investments (for any trade partners) will contribute to our understanding on the ambiguous results so far. In theory, a dynamic model for bilateral economy with rich firm-level heterogeneity will lay more promising micro foundation. For instance, although exchange rate movement will be same for all firms the market demand will not necessarily be same because different industries have different specific uncertainties in their products' domain. Additionally, data from firm-level investments can provide more observations on lumpy investment spikes, which should be much closer to our real options setting instead of the data at country-level. Our paper also calls for the new direction in the application of Machine Learning to continuous-time models. Duarte et al. (2024) developed a deep policy learning algorithm for solving nonlinear high-dimensional continuous-time models in the fields of asset pricing, corporate finance and portfolio choice. In our classic finite difference method, the PDE is solved slowly due to iteration and approximation. The new deep learning algorithm could largely decrease computation and simulation time and incorporate more stochastic variables such as interest rate and inflation rate, both of which can also impact foreign exchange. Therefore, application of machine learning can embrace more economic factors and makes the results closer to the real economy.

Appendix A: Empirical Exercises for Section 3

The FDI data come from BEA and shows U.S. Direct Investment Abroad. BEA obtains these data from comprehensive mandatory surveys of U.S. multinational firms, and are available in considerable detail by country. Foreign direct investment abroad is defined by the BEA as ownership by a U.S. investor of at least 10% of a foreign business and measures the total outstanding level of U.S. direct investment abroad at yearend. Similarly, information on inward FDI into the U.S. is obtained from mandatory reporting by all U.S. business enterprises in which a foreign firm or individual owns, directly or indirectly, 10% or more of the voting securities of an incorporated U.S. business enterprise or an equivalent interest of an unincorporated U.S. business enterprise.

We use GDP growth volatility as a proxy for market demand. In previous literature, scholars have used GDP growth as demand proxy, for instance Goldberg and Kolstad (1995). However, the standard deviation of GDP growth for most countries is very trivial. We use GDP growth volatility instead. We obtain the GDP growth rate from the World Banks databank. We define the demand volatility as the five-year rolling standard deviation of GDP growth.

We include both the exchange rate return and the volatility in exchange rate returns as independent variables. Exchange rate is defined as the *number of foreign currencies per USD*. This definition remains the same whether U.S. is the home or host country. The volatility in exchange rate returns is defined as the standard deviation of exchange rate returns. The main independent and dependent variables are Winsorized for outliers at 99%. Other controls in the estimation include GDP growth rate, FDI as a percent of GDP and the level of inflation. We also include year and country controls. The data for all the controls are obtained from World Bank's World Development Indicators.

We include the correlation between the two volatilities—exchange rate returns and foreign demand growth. To obtain the correlation, we first estimate quarterly GDP growth volatilities and then obtain annual correlation between the volatility of exchange rate and the GDP growth volatility.

The table below provides the data summary of all the variables used in the estimation.

We test for stationarity in the main independent variables and the dependent variable using the Augmented Dickey Fuller test. We find the presence of unit root in the GDP growth series, and both the volatilities for some countries. To address the nonstationarity, we include first difference of the series where there is unit root.

Regression Analysis

Our estimation strategy is very similar to Goldberg and Kolstad (1995). However, we expand our study to seven countries instead of just two and study the data for a longer period of time using annual data instead of quarterly data. One caveat in our data is that we cannot identify the type of FDI—horizontal or vertical. This issue may be unavoidable since most MNCs engage in some combination of horizontal and vertical FDI. However, according to Fillat et al. (2015) most foreign sales are horizontal. We acknowledge that this caveat may cast some constraints on the explaining power of the regression.

Our reduced form equation that measures the relationship between FDI flow into the host country and the demand and exchange rate volatilities is given below.

$$\text{FDI}_{i,j,t} = \beta_0 + \beta_1 \text{ER Returns}_{j,t} + \beta_2 \text{ER Volatility}_{j,t} + \beta_3 \text{GDP Growth Volatility}_{j,t} + \beta_4 \text{Correlation}_{j,t} + \gamma_j + \tau_i + \epsilon_{i,j,t}$$

FDI flow is log linearized which leaves us with only the positive values of FDI flow.[7] $\text{FDI}_{i,j,t}$ shows the flow of FDI from country i to country j in time t. We include both home country controls and destination country controls. The estimation is an OLS regression run for each pair of countries. Since we do not have the FDI inflow information between the seven countries, we do

Table A.1: Summary of variables.

	Minimum	Maximum	Mean	25th Percentile	Median	75th Percentile
FDI Flow	−0.051	0.195	0.001	0.000	0.000	0.001
ER Returns	−0.294	0.223	−0.004	−0.061	−0.001	0.050
ER Volatility	0.009	0.180	0.082	0.059	0.079	0.102
GDP Growth Volatility	0.149	4.722	1.632	0.885	1.452	2.065
Correlation	−1.000	1.000	0.008	−0.850	−0.043	0.888
FDI as percent of GDP	−26.195	86.589	2.582	0.732	1.424	2.330
Inflation	−2.294	12.091	2.164	1.221	1.994	2.903
GDP growth rate	−5.619	7.259	2.366	1.567	2.518	3.678

[7] We also redo the regression with full FDI sample and the results are similar.

not use a panel estimation or a pooled regression. Again here we follow Goldberg and Kolstad (1995) and conduct pairwise estimation of the relationship. To facilitate our presentation, the summary of the estimation result with bilateral estimations is given below

Table A.2: OLS regression of log FDI Flow on exchange rate volatility and GDP growth volatility (with U.S. as the destination country).

Variables	(1) U.K.	(2) Japan	(3) Canada	(4) Germany	(5)† Netherlands	(6) Switzerland	(7) France
ER Returns	7.598*	5.353**	12.088	−4.968*	4.153	13.179**	7.689***
	(3.442)	(2.457)	(10.196)	(2.409)	(2.773)	(5.594)	(1.400)
ER Volatility	8.186	15.883	−9.642	−12.927	11.038	−16.743	−7.814
	(13.447)	(10.338)	(34.069)	(7.338)	(27.707)	(16.004)	(6.513)
GDP Growth Volatility	−0.155	−0.035	0.227	−0.769**	0.376	0.533	0.008
	(0.352)	(0.333)	(0.552)	(0.302)	(0.484)	(0.374)	(0.273)
Correlation	−0.477	−0.114	−0.435	0.560*	−0.235	0.294	−0.449*
	(0.373)	(0.397)	(0.473)	(0.288)	(0.576)	(0.503)	(0.247)
Constant	−6.175	−9.368***	−6.860**	−0.745	−7.614	−5.688*	−6.644***
	(3.509)	(2.206)	(2.548)	(3.055)	(4.615)	(2.563)	(1.183)
Observations	17	22	23	20	16	18	22
R^2	0.417	0.709	0.308	0.641	0.663	0.624	0.856

Dependent variable is logarithm of FDI Flow. The data for the estimation run from 1982 to 2018. ER Returns is the difference between the average annual exchange rate of one year from the previous year. ER Volatility is the volatility of the exchange rate estimated as the standard deviation of exchange rate returns. GDP Growth Volatility is the volatility of the GDP growth rate estimated as the standard deviation of GDP growth rate. Correlation is the correlation between GDP Growth Volatility and exchange rate returns volatility (the volatilities estimated from quarterly and monthly data, respectively). It is the correlation between country pairs—which means the correlation changes over years and over country pairs. The estimation includes FDI as a percent of GDP, one period lagged inflation, GDP, and GDP growth rate as controls. Lagged GDP volatility and exchange rate returns volatility are included where necessary to correct for unit root. † Includes lagged dependent variable to correct for unit root.
Robust standard errors in parentheses.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table A.3: This table is similar to Table A.2 except here U.S. is the home country.

Variables	(1) U.K.	(2) Japan	(3) Canada	(4) Germany	(5) Netherlands	(6) Switzerland	(7) France
ER Returns	−2.009	3.637*	1.822	−10.437	−2.291	−8.928*	−1.143
	(7.351)	(1.645)	(21.699)	(5.860)	(3.180)	(3.804)	(2.223)
ER Volatility	8.478	14.244**	20.072	−20.011	8.697	−23.318**	2.496
	(28.699)	(4.501)	(79.174)	(14.511)	(7.787)	(8.677)	(7.450)
GDP Growth Volatility	0.067	−1.103***	0.432	0.250	−0.172	−1.812**	−0.387
	(1.222)	(0.234)	(0.830)	(0.398)	(0.536)	(0.614)	(0.599)
Correlation	−0.606	0.235	0.248	−0.431	−0.805	0.589	−0.289
	(0.936)	(0.258)	(0.508)	(0.434)	(0.732)	(0.358)	(0.367)
Constant	−10.881	−4.232*	−9.695*	−6.915**	−9.023***	−5.915*	−7.612***
	(10.576)	(1.975)	(3.904)	(2.338)	(2.431)	(2.610)	(1.294)
Observations	16	20	15	16	21	17	16
R^2	0.335	0.814	0.211	0.637	0.430	0.849	0.423

Robust standard errors in parentheses.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Appendix B: Derivation of Equations (3) and (8)

Derivation of Equation (3)

Let us start with a general process

$$\frac{dS}{S} = \mu_S dt + \sigma_S dW^Q \tag{B.1}$$

We will prove $\mu_S = r_h - r_f$ under the risk-neutral measure, given S is denominated by the home currency value of foreign currency. Let H_t and F_t denote the share prices of the assets in the home money market and foreign money market, reported in units of home currency and foreign currency, respectively, and normalized so that the time-zero share prices are both 1. Then

$$H_t = exp(r_h t) \text{ and } F_t = exp\left(r_f t\right)$$

The share price of foreign money market at time t in home currency is $F_t S_t$. Solving the stochastic differential Equation (B.1) gives the explicit formula

$$F_t S_t = S_0 exp\left\{\left(r_f + \mu - \frac{1}{2}\sigma_S{}^2\right)t + \sigma_S W_t\right\} \tag{B.2}$$

Note the present value of share price at home currency is

$$exp(-r_h t)F_t S_t = S_0 exp\left\{\left(r_f - r_h + \mu\right)t\right\}exp\left\{-\frac{1}{2}\sigma_S{}^2 t + \sigma_S W_t\right\} \tag{B.3}$$

Since under $\mathbb{Q}$ measure, the discounted share price of the asset foreign money market in home currency must be a martingale, also note that the second exponential term is a martingale by itself, to see this, let's define function as $U(t) = exp\left\{-\frac{1}{2}\sigma^2 t + \sigma W_t\right\}$ with information filter $(F_t)_{t\geq 0}$:

$$\begin{aligned} E(U(t+s)|F_s) &= E\left(exp\left\{-\sigma^2(t+s)/2 + \sigma W_{t+s}\right\}|F_s\right) \\ &= exp\left\{-\sigma^2 s/2 + \sigma W_s\right\}E\left(exp\left\{-\sigma^2 t/2 + \sigma(W_{t+s} - W_s)\right\}|F_s\right) = U(s) \end{aligned}$$

Note that the above short proof requires condition $E(exp\{\sigma W\}) = exp\left\{\sigma^2/2\right\}$ which is established when W is a normal distribution with $N(0,\ \sigma^2)$. Thus, we obtain $\mu_S = r_h - r_f$

(Q.E.D.)

Derivation of Equation (8)

There are several approaches to derive this PDE. Here we follow a heuristic manner. Assume that the risks inherent in exchange rate S and foreign market demand θ are spanned by the market of existing securities. Let call these securities "currency" and "demand" for brevity. Recall that in a risk-neutral world, the instantaneous return of holding foreign assets V_F is equal to risk-free rate

$$E\left[\frac{dV}{V}\right] + \frac{\pi}{V} = r_f \tag{B.4}$$

The Itô's lemma says that $V = V(S, \theta)$ obeys

$$\begin{aligned} dV =& \left[\frac{1}{2}\sigma_S{}^2 S^2 \frac{\partial^2 V_F}{\partial S^2} + \frac{1}{2}\sigma_\theta{}^2\theta^2\frac{\partial^2 V_F}{\partial\theta^2} + \rho\sigma_S\sigma_\theta S\theta\frac{\partial^2 V_F}{\partial S\partial\theta} + \left(r_h - r_f\right)S\frac{\partial V_F}{\partial S} + \mu\theta\frac{\partial V_F}{\partial\theta}\right]dt \\ &+ {}_S S\frac{\partial V_F}{\partial S}dW + \sigma_\theta\theta\frac{\partial V_F}{\partial\theta}dZ \end{aligned}$$

Since the expectation of the last two items is zero, substitute it into Equation (B.4) we obtain Equation (8).

Appendix C: Numerical Methods

We use finite difference method to solve the equation. The mesh grid takes the form of centered discretization scheme and is constructed as (θ_i, S_j) with $i = 1, 2\ldots.n$ and $j = 1, 2\ldots.m$, where $\theta_i = 0, \Delta\theta, 2\Delta\theta\ldots\ldots n\Delta\theta$ and $S_j = 0, \Delta S, 2\Delta S\ldots\ldots m\Delta S$, applying Tylor expansion in the error order of $O(h^2)$

$$\frac{\partial V}{\partial S} = \frac{V_{i,j+1} - V_{i,j-1}}{2\Delta S}$$

$$\frac{\partial V}{\partial \theta} = \frac{V_{i+1,j} - V_{i-1,j}}{2\Delta\theta}$$

$$\frac{\partial^2 V}{\partial \theta^2} = \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta\theta)^2}$$

$$\frac{\partial^2 V}{\partial S^2} = \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{(\Delta S)^2}$$

$$\frac{\partial^2 V}{\partial S \partial \theta} = \frac{V_{i+1,j+1} - V_{i+1,j-1} - V_{i-1,j+1} + V_{i-1,j-1}}{4\Delta S \Delta\theta}$$

After applying the above equations we have

$$a_i V_{i-1,j} + b_{i,j} V_{i,j} + c_i V_{i+1,j} + d_j V_{i,j-1} + e_j V_{i,j+1} + f_{i,j}\left(V_{i+1,j+1} - V_{i+1,j-1} - V_{i-1,j+1} + V_{i-1,j-1}\right) = -\pi_X$$

with the following six definitions

$$a_i = \frac{\sigma_\theta{}^2\theta^2}{2(\Delta\theta)^2} - \frac{\mu\theta}{2\Delta\theta}; \quad b_{i,j} = -\frac{\sigma_\theta{}^2\theta^2}{(\Delta\theta)^2} - \frac{\sigma_S{}^2 S^2}{(\Delta S)^2} - r_h; \quad c_i = \frac{\sigma_\theta{}^2\theta^2}{2(\Delta\theta)^2} + \frac{\mu\theta}{2\Delta\theta}$$

$$d_j = \frac{\sigma_S{}^2 S^2}{2(\Delta S)^2} - \frac{(r_h - r_f)S}{2\Delta S}; \quad e_j = \frac{\sigma_S{}^2 S^2}{2(\Delta S)^2} + \frac{(r_h - r_f)S}{2\Delta S}; \quad f_{i,j} = \frac{\rho\sigma_S S \sigma_\theta \theta}{4\Delta S \Delta\theta}$$

The boundary conditions are set as $V_{1,j} = 0$ and $V_{i,1} = 0$

To start the iteration we first set an initial guess $V_{i,j}^0 = max\left[V_X^0, V_F^0 - I, 0\right]$, where V_X^0, V_F^0 are the present value of all operating profit flows given the capacity is in operation

$$V_X^0(\theta, S) = E^Q \int_t^\infty e^{-r_h(s-t)} \pi_X ds = \frac{1}{4\gamma}\left(\frac{\tau S\theta^2}{r_f - 2\mu - \sigma_\theta{}^2 - 2\rho\sigma_S\sigma_\theta} - \frac{2v_h\theta}{r_h - \mu} + \frac{v_h{}^2}{\tau S(2r_h - r_f)}\right)$$

$$V_F^0 = E^Q \int_t^\infty e^{-r_h(s-t)} \pi_F ds = \frac{S}{4\gamma}\left(\frac{\theta^2}{r_f - 2\mu - \sigma_\theta{}^2 - 2\rho\sigma_S\sigma_\theta} - \frac{2v_f\theta}{r_f - \mu - \rho\sigma_S\sigma_\theta} + \frac{v_f{}^2}{r_f}\right)$$

Note that there is a series of limitation that all denominators have to be positive so that there is no assets bubble, we concatenate all the limits to the following requirement

$$2r_h > r_f > 2\mu + \sigma_\theta{}^2 + 2\rho\sigma_S\sigma_\theta$$

Then the $V_{i,j}^{iter>0}$ is computed for each iteration and the computation will be ceased at the tolerance $V_{i,j}^{iter+1} - V_{i,j}^{iter} < \varepsilon$, where ε is the extremely small number. At the terminal knot since we assume *R*(*j*) and *D*(*i*) are sufficiently large we assume the 3-order derivative impact at $V_{n-1,m-1}$ is negligible

$$\frac{\partial^3 V_{n-1,\ 2:n}}{\partial \theta_{n-1}{}^3} = \frac{V_{i+2,j} - 2V_{i+1,j} + 2V_{i-1,j} - V_{i-2,j}}{2(\Delta\theta)^3} = 0$$

Thus, we have

$$V_{n+1,j} = 2V_{n,j} - 2V_{n-2,j} + V_{i-3,j}$$

$$\frac{\partial^3 V_{[2:m],\ m-1}}{\partial S_{m-1}{}^3} = \frac{V_{i,j+2} - 2V_{i,j+1} + 2V_{i,j-1} - V_{i,j-2}}{2(\Delta S)^3} = 0$$

Thus, we have

$$V_{i,m+1} = 2V_{i,m} - 2V_{i,m-2} + V_{i,m-3}$$

The system is then solved using the method of successive over-relaxation (SOR), a variant of the Gauss-Seidel method, which is a method for solving linear systems of equations. The SOR method is an iterative finite difference method that includes a relaxation factor $1 < \omega < 2$ with purpose being to accelerate convergence and we set it $\omega = 1.2$.

Acknowledgments: *Michi Nishihara was supported by the JSPS KAKENHI (Grant numbers JP20K01769, 23K20613, JP24K00272).*

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